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Guidance for Asteroid Rendezvous

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Introduction

THE use of electric propulsion at very low thrust levels for different missions to asteroids and comets is a very attractive option. Various aspects of the guidance and navigation systems for these missions were studied previously.[1-4]

The rendezvous with a celestial object has a number of phases: 1) cruise or transfer to the vicinity of the celestial body, 2) approach, and 3) maneuvers near the celestial body. This Note is concerned with the approach phase.

In Ref. 4, an onboard autonomous guidance system was proposed based on a steerable platform carrying an imaging system that points at the celestial body measuring the line-of-sight (LOS) inertial rate and a laser system that is aligned with the imaging system and measures the relative range to the celestial body.

For a spacecraft with an autonomous onboard tracking system, the relative velocity components along and normal to the LOS as well as the relative range are the most adequate inputs for a guidance law. The use of LOS coordinates for the guidance law was considered previously in Ref. 5, where the commanded acceleration along the LOS was defined as a function of the square root range, and for the accelerations normal to the LOS, two different cases were considered: 1) proportional to the LOS rate as in proportional navigation[6] or 2) proportional to the relative velocity normal to the LOS.

In this work, a guidance law in LOS coordinates, which is a generalization of the exponential type of guidance studied in Ref. 7, is considered for the rendezvous with an asteroid.

System Definition

Figure 1 depicts the in-plane rendezvous geometry, where S is an active spacecraft able to apply a continuous thrust controlled both in amplitude and direction, and A is a passive object, a celestial body of small dimensions.

Spacecraft rendezvous maneuvers in the vicinity of the celestial object will be considered. The duration of these maneuvers relative to the celestial body orbital period being negligible, a constant rectilinear motion for the celestial body will be assumed.

Using an inertial system of coordinates centered on A, the relative equations of motion are defined by

$$R\ddot{\theta} + 2\dot{R}\dot{\theta} = a_\theta \tag{1}$$

$$\ddot{R} - R\dot{\theta}^2 = a_r \tag{2}$$

where R, $\dot{R}$, and $\ddot{R}$ are the relative SA range, range rate, and range acceleration, respectively; $\dot{\theta}$ and $\ddot{\theta}$ are the LOS angular rate and acceleration; and a_r and a_θ are the acceleration components along and normal to the LOS acting on the S vehicle.

The use of autonomous guidance systems for interception is widely known. One of the simplest—and probably the most successful—of the guidance laws for interception is proportional navigation, where the intercepting vehicle maneuvers normal to the LOS are made proportional to its angular rate. This assures that the LOS rate approaches zero for nonmaneuvering targets, a collision course is achieved, and a successful interception is ensured.

While interception is the matching of the position vectors of two vehicles, the rendezvous of two vehicles is the simultaneous matching of both their position and velocity vectors. The use of the LOS rate, or possibly the relative velocity normal to the LOS, is only part of the rendezvous problem, since the relative velocity must be zero when the two vehicles meet. The active vehicle acceleration then must include a component along the LOS that reduces the relative closing velocity to zero as the two vehicles approach one another.

A guidance rendezvous law of the form

$$a_{\theta c} = c_1 R\dot{\theta} \tag{3}$$

$$a_{rc} = c_2\dot{R} - c_3 R \tag{4}$$

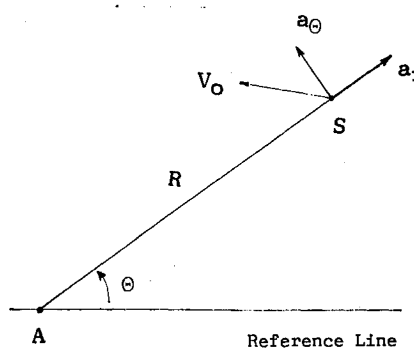

Fig. 1 Relative geometry.

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where c_1, c_2, and c_3 are three positive constants, is considered for analysis.

For the particular case $c_1 = c_2$, this is the exponential type of guidance studied in Ref. 7, where vehicle accelerations were made proportional, in rectangular coordinates, to the components of range and range rate.

For the initial approach ranges, on the order of a few thousand kilometers, the gravitational effects due both to the asteroid and the gradient of the heliocentric field can be assumed to be negligible.[4] It follows then that the accelerations acting on the vehicle S are equal to those defined in Eqs. (3) and (4).

Substituting Eqs. (3) and (4) into Eqs. (1) and (2), the following nonlinear system of equations is obtained:

$$R\ddot{\theta} + 2\dot{R}\dot{\theta} = c_1 R\dot{\theta} \tag{5}$$

$$\ddot{R} - R\dot{\theta}^2 = -c_2\dot{R} - c_3 R \tag{6}$$

Determination of Trajectories

The system trajectories as defined by the nonlinear differential equations (5) and (6) now will be considered for analysis.

Multiplying Eq. (5) by R, rearranging, and solving, with initial conditions R_0, $\dot{\theta}_0$ at $t = 0$, θ is obtained as

$$\dot{\theta} = \dot{\theta}_0 e^{-c_1 t}(R_0/R)^2 \tag{7}$$

Squaring Eq. (7), substituting into Eq. (6), and rearranging,

$$\ddot{R} + c_2\dot{R} + [c_3 - \dot{\theta}_0{}^2 e^{-2c_1 t}(R_0/R)^4]R = 0 \tag{8}$$

Let us now define a new variable x such that,

$$R = R_0\sqrt{\dot{\theta}_0}\, e^{-c_1 t/2} x \tag{9}$$

Substituting R from Eq. (9) and its derivatives with respect to t into Eq. (8) gives

$$\ddot{x} + a_1\dot{x} + a_2 x - x^{-3} = 0 \tag{10}$$

where

$$a_1 = c_2 - c_1 \tag{11}$$

$$a_2 = (c_1^2 - 2c_1c_2 + 4c_3)/4 \tag{12}$$

The variable x is defined by an autonomous nonlinear second-order differential equation, and the solution of this equation will provide the trajectories of the system.

Equation (10) can be solved in closed form only for the particular case $a_1 = 0$, or, equivalently, $c_1 = c_2$. In the general case, no closed-form solution can be obtained, and in order to study the x solution, qualitative methods[8] of analysis in the phase plane are employed.

Depending on the system gains c_1, c_2, and c_3, five different cases can be distinguished.

Case 1:

$$c_1^2 - 2c_1c_2 + 4c_3 < 0$$

$x(t)$ behaves as an exponentially increasing function of time,

$$x(t) \simeq e^{c_0 t}$$

where

$$c_0 = \left[c_1 - c_2 + \sqrt{(c_2^2 - 4c_3)}\right]/2 \tag{13}$$

i.e., c_0 is the type number of $x(t)$. [Lyapunov type numbers (Ref. 8, pp. 50-55): If $f(t)$ is any continuous function of t, and α is any real number such that $f(t)e^{\alpha t}$ is bounded in $(0, +\infty)$, then $f(t)e^{\alpha' t}$ with $\alpha' < \alpha$ is also bounded in $(0, +\infty)$ and approaches zero at $t \to +\infty$. If $f(t)e^{\beta t}$ is unbounded in $(0, +\infty)$, then $f(t)e^{\beta' t}$ with $\beta' > \beta$ is also unbounded in $(0, +\infty)$. Thus, if A and B are the classes of all real numbers α, β with $(f)\, te^{\alpha t}$ bounded and $f(t)e^{\beta t}$unbounded in $(0, +\infty)$, then (A,B) is a partition of the real field defining a real number δ said to be the *type number* of $f(t)$ in $(0, +\infty)$.]

Case 2:

$$c_1^2 - 2c_1c_2 + 4c_3 > 0$$

$$c_2^2 > 4c_3$$

$$c_2 < c_1$$

In this case, $x(t)$ is also an exponentially increasing function of time with the same type number c_0 as in case 1.

Case 3:

$$c_1^2 - 2c_1c_2 + 4c_3 > 0$$

$$c_2^2 > 4c_3$$

$$c_2 > c_1$$

The nonlinear equation has a stable singular point,

$$\lim_{t\to\infty} x(t) = a_2^{-1/4} \tag{14}$$

$$\lim_{t\to\infty} \dot{x}(t) = 0 \tag{15}$$

Case 4:

$$c_2^2 < 4c_3$$

$$c_2 > c_1$$

$x(t)$ approaches the same stable singular point in an oscillatory way.

Case 5:

$$c_2^2 < 4c_3$$

$$c_2 < c_1$$

$x(t)$ oscillates between a minimum value approaching zero and an exponentially increasing maximum value.

The five different cases are depicted in Fig. 2 in the $c_1 - c_2$ plane.

The intersection point

$$c_1 = c_2 = 2\sqrt{c_3} \tag{16}$$

corresponds to the so-called critical damping case.[7]

Based on relation (9) and the behavior of $x(t)$, it follows that, for any positive set of values c_1, c_2, and c_3,

$$\lim_{t\to\infty} R(t), \dot{R}(t) = 0 \tag{17}$$

Substituting x by R into Eq. (7) and rearranging,

$$\dot{\theta} = \text{sign}(\dot{\theta}_0)x^{-2} \tag{18}$$

For $\dot{\theta}(t)$, three basic behavior patterns can be distinguished.

I) For cases 1 and 2,

$$\lim_{t\to\infty}\dot{\theta}(t) = 0 \tag{19}$$

II) For cases 3 and 4,

$$\lim_{t\to\infty}\dot{\theta}(t) = \text{sign}(\dot{\theta}_0)\sqrt{a_2} \tag{20}$$

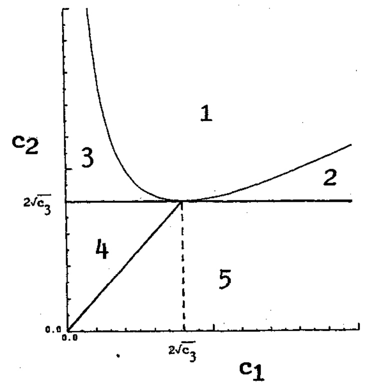

Fig. 2 Gains map.

III) For case 5, $\dot{\theta}(t)$ is an oscillatory diverging function of time.

Spiraling Approach

Based on the analysis previously performed, the guidance gains c_1, c_2, and c_3 will be defined in order to obtain a predetermined trajectory type.

Generally, the basic criterion for rendezvous guidance synthesis was to achieve trajectories either with minimum time or minimum energy expenditure.[9] This is not the approach adopted herein. For the case of a small celestial object with an irregularly shaped body, unknown gravitational constant, and probably spinning at an unknown rate, a particular trajectory that provides time for the object analysis prior final approach can certainly be useful. Moreover, a trajectory that can ensure a direct injection into a parking orbit has a definite advantage over other approaches. It will now be shown that an adequate choice of gains c_1, c_2, and c_3 will ensure these requirements.

For a rendezvous trajectory that initiates at

$$x(0) = a_2^{-1/4}, \qquad \dot{x}(0) = 0 \tag{21}$$

it follows that

$$x(t) = a_2^{-1/4}, \qquad \dot{x}(t) = 0 \quad \text{for all} \quad t \geq 0 \tag{22}$$

In this case, following from Eqs. (19) and (18),

$$R(t) = R_0 e^{-c_1 t/2} \tag{23}$$

$$\dot{\theta}(t) = \text{sign}(\dot{\theta}_0)\sqrt{a_2} \tag{24}$$

The trajectory is an exponentially decreasing spiral with a constant angular rate.

To ensure this type of trajectory, gains c_1, c_2, and c_3 are to be defined such that at rendezvous initiation the $x(t)$ initial conditions are at the singular point, as defined in Eq. (22). From Eq. (22) it follows that the system gains verify

$$c_1^2 - 2c_1c_2 + 4c_3 = 4\dot{\theta}_0^2 \tag{25}$$

$$c_1 = -2\dot{R}_0/R_0 \tag{26}$$

With these gains, the spacecraft commanded acceleration amplitude and direction are defined by

$$a_T = \sqrt{a_{\theta c}^2 + a_{rc}^2} = (V_0^2/R_0)e^{\dot{R}_0 t/R_0} \tag{27}$$

$$\Phi_a = a\tan(a_{\theta c}/a_{rc}) = 2\Phi_{V0} \tag{28}$$

where V_0 and Φ_{V_0} are the initial spacecraft relative velocity amplitude and direction, respectively. The required acceleration has an exponentially decreasing amplitude, with a maximum at rendezvous initiation, and a constant direction with respect to the LOS.

Orbit Injection

At very close ranges, the gravitational effect of an asteriod with a diameter ranging in the tens of kilometers influences the spacecraft trajectory.

For instance, at a range of 200 km, for a probe with a mass of 900 kg and an ion thruster with a maximum thrust of 100 mN,[4] the gravitational acceleration of an asteroid with

$$\mu = GM = 3900\ \text{km}^3/\text{h}^2$$

where μ is the gravitational constant G times the mass of the asteroid M, becomes comparable in magnitude to the applied acceleration due to the thrust forces.

To ensure the possibility of estimating both the gravitational characteristics and spin rates of the asteroid, a parking orbit is suggested. To ensure injection to an elliptic orbit, the following condition is to be met[10]:

$$E = V^2/2 - \mu/R < 0 \tag{29}$$

For a spiraling approach trajectory, this condition implies that

$$R < [2\mu(R_0/V_0)^2]^{1/3} \tag{30}$$

This expression defines the maximum range at which injection can be performed by simply *switching off the thruster*.

To avoid a collision with the asteroid, an additional condition is to be imposed on the parking orbit; namely, the required minimum value of periapsis should be greater than the maximum asteroid radius. In other terms,

$$a(1 - e) > R_A \tag{31}$$

where R_A is the maximum asteroid radius, a the semimajor axis, and e the orbit eccentricity.

For the spiraling trajectory, it follows from Eq. (31), by adequate algebraic manipulation, that in order to avoid collision with the asteroid in the parking orbit, the spacecraft velocity at rendezvous initiation should verify

$$V_{\theta 0} = R_0\dot{\theta}_0 > \sqrt{R_A}\,(V_0^8/2\mu R_0^2)^{1/6} \tag{32}$$

An example of rendezvous trajectory and injection into a parking orbit is shown in Fig. 3.

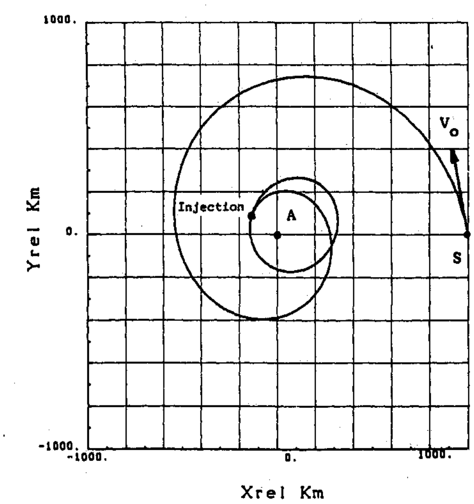

Fig. 3 Spiral trajectory and orbit injection.

Spacecraft initial conditions are

$$R_0 = 1000\,\text{km}, \qquad V_{r0} = -5\,\text{km/s}, \qquad V_{\theta 0} = 25\,\text{km/s}$$

and the spacecraft guidance gains are

$$c_1 = 0.01, \qquad c_2 = 0.06, \qquad c_3 = 0.0009$$

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Stabilization via Dynamic Output Feedback: A Numerical Approach

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I. Introduction

IN the design of controllers, the complete state is often not available for feedback. One can reconstruct the state using a Luenberger observer,[1] but this may unduly complicate the structure of the controller due to the large dimension of the observer dynamics. (If n is the dimension of the state and p the number of measured states, then the observer will be of order n-p.) In Ref. 2, a method is presented for determining constant output feedback gains for the control of systems with inaccessible states. The control is linearly dependent on the measured outputs. However, it may not be possible to achieve stabilization with constant output feedback; a spring-mass system with position as output is the classic example.

Here, we present a numerical method for determining a dynamic output feedback controller where the controller order is less than the (n-p)th order controller required in Ref. 1. Such a controller has the capability of stabilizing systems where static output feedback fails while being simpler in structure than the control based on the separation theorem, where one first designs the controller gains assuming the complete state can be measured and then uses the same gains with an estimated state. The procedure is extended to the problem of simultaneously stabilizing several plants by a single controller.

II. Problem Formulation

Consider a linear, time-invariant system described by

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad y(t) = Cx(t) \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, and $y(t) \in R^p$ are the state, control, and measured output, respectively. It is assumed that the system is controllable and observable.

Static output feedback problem: Determine an $m \times p$ matrix K such that

$$u(t) = Ky(t) \tag{2}$$

stabilizes Eqs. (1); i.e., the eigenvalues of $A + BKC$ lie in the left half of the complex plane.

Dynamic output feedback problem: Let $z(t) \in R^q$ satisfy

$$\dot{z}(t) = Fz(t) + Gy(t) \tag{3}$$

Determine the dimension q and the matrices $K_1 \in R^{m\times q}$ and $K_2 \in R^{m\times p}$ such that

$$u(t) = K_1 z(t) + K_2 y(t) \tag{4}$$

stabilizes Eqs. (1) and (3).

For the static output feedback problem, it is shown in Ref. 3 that max $\{m,p\}$ eigenvalues can be assigned. Thus, in general, stability cannot always be achieved via static output feedback because nothing can be done about the remaining $n - \max\{m,p\}$ eigenvalues. However, it may still be possible to stabilize the system using output feedback, but the possibility of accomplishing this must be ascertained on a problem by problem basis; no general theory is available.

For the dynamic output feedback problem, one question that arises is the choice of the smallest dimension q of the dynamic compensator needed to achieve stability. There is no theoretical result specifying the minimum order of the compensator required for pole assignability (or even stabilizability). In Ref. 4, it is shown that this order is at most equal to $\min\{\rho_c, \rho_o\}$, where ρ_c and ρ_o are the controllability and observability indices, respectively, defined by

$$\rho_c = \min\{\rho:\ \text{rank}[B \quad AB \quad \dots \quad A^\rho B] = n\}$$

$$\rho_o = \min\{\rho:\ \text{rank}[C' \quad A'C' \quad \dots \quad (A')^\rho C'] = n\}$$

Kimura[5] states that if the order q of the dynamic compensator is chosen to be $q = n - m - p + 1$ then arbitrary eigenvalue assignment is possible. Both of these results are sufficient conditions for arbitrary eigenvalue assignment; it may be possible to achieve eigenvalue assignment or stability by a compensator of lower order than required by either theory.

In the next section, we review the numerical technique of Ref. 2 for solving the static output feedback problem and give some modifications of the technique that improves convergence. This procedure involves the minimization of a suitable objective function. Then, we show that the dynamic output

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